The Gradient Operator in Coordinate Systems

For the **Cartesian** coordinate system, the Gradient of a scalar field is expressed as:

$$\nabla g(\bar{r}) = \frac{\partial g(\bar{r})}{\partial x}\hat{a}_x + \frac{\partial g(\bar{r})}{\partial y}\hat{a}_y + \frac{\partial g(\bar{r})}{\partial z}\hat{a}_z$$

Now let's consider the gradient operator in the **other** coordinate systems.

Q: *Pfft! This is easy! The gradient operator in the spherical coordinate system is:*

$$\nabla g(\bar{\text{r}}) = \frac{\partial g(\bar{\text{r}})}{\partial r}\hat{a}_r + \frac{\partial g(\bar{\text{r}})}{\partial \theta}\hat{a}_\theta + \frac{\partial g(\bar{\text{r}})}{\partial \phi}\hat{a}_\phi$$

Right ??

A: **NO!!** The above equation is **not** correct!

Instead, we find that for **spherical** coordinates, the gradient is expressed as:

$$\nabla g(\bar{r}) = \frac{\partial g(\bar{r})}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial g(\bar{r})}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial g(\bar{r})}{\partial \phi}\hat{a}_\phi$$

And for the **cylindrical** coordinate system we likewise get:

$$\nabla g(\bar{r}) = \frac{\partial g(\bar{r})}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial g(\bar{r})}{\partial \phi}\hat{a}_\phi + \frac{\partial g(\bar{r})}{\partial z}\hat{a}_z$$